Training Material for Balances (2)

Prepared by R&D Division 1

A&D Company, Limited

Contents

Measuring *Repeatability*, *Linearity & Sensitivity Drift*

Specifications of GX-2000

Weighing Capacity	2100g
Minimum Weighing Value (1 digit)	0.01g
Repeatability (Standard Deviation)	0.01g
Linearity	±0.02g
Sensitivity Drift (10 ~ 30ºC)	±2ppm/ºC (Automatic self calibration is not used)

A&D Company, Limited

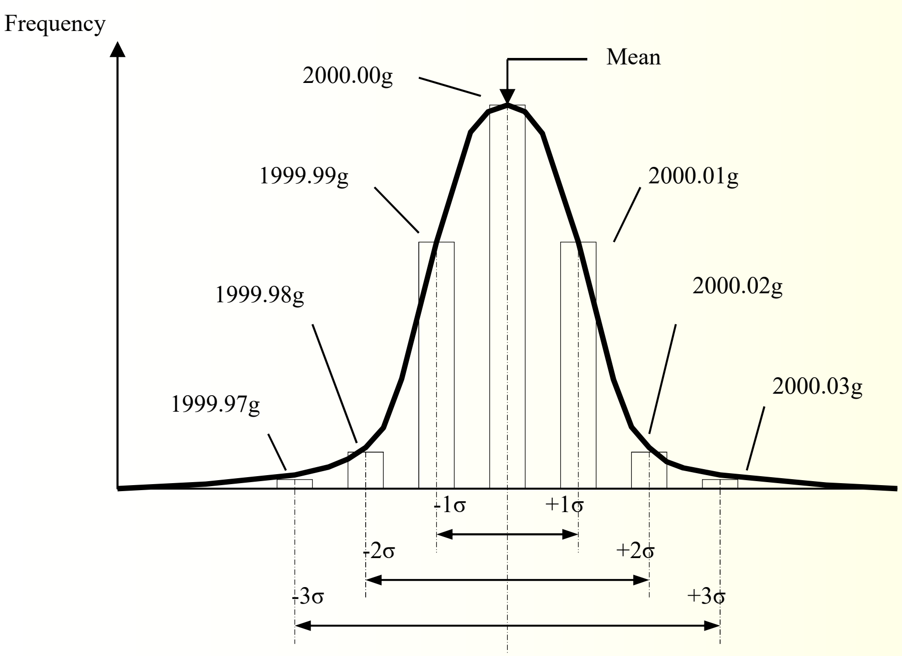
Repeatability

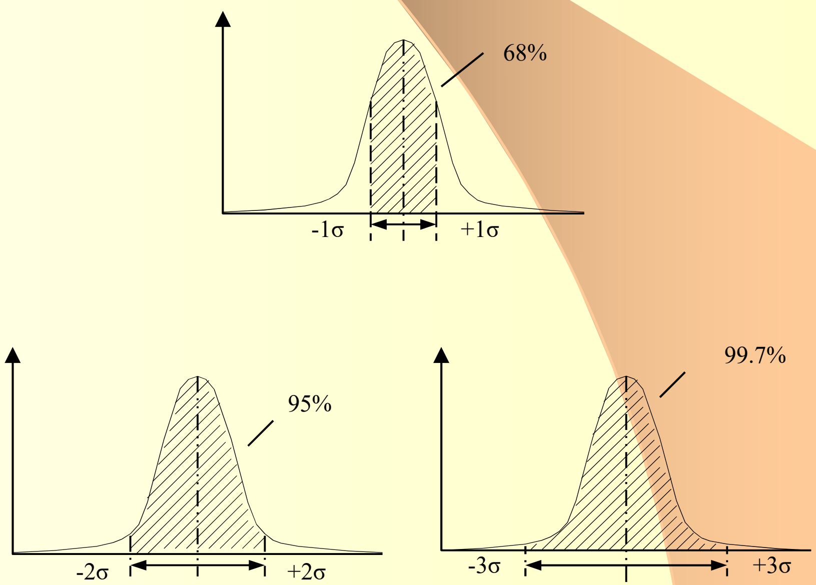
- Repeatability indicates the variation in measurement when the same mass is loaded several times. Repeatability is expressed using standard deviation (σ: sigma).
- As the repeatability of GX-2000 is 0.01g, when a 2kg mass is loaded repeatedly, the results will be*
  - 1999.99g ~ 2000.01g with the probability of 68% (Mean $\pm 1\sigma$)
  - 1999.98g ~ 2000.02g with the probability of 95% (Mean $\pm 2\sigma$)
  - 1999.97g ~ 2000.03g with the probability of 99.7% (Mean $\pm 3\sigma$)

* supposing that the mean value is 2000.00g

Repeatability

Normal Distribution





Repeatability

Calculating Repeatability (σ)

Formula: $$\text{Standard Deviation } \sigma_{n-1} = \sqrt{\frac{\sum(\text{Measured Value} - \text{Mean})^2}{\text{Number of Measurements} - 1}}$$

Ex) We measured a 2kg mass five times with GX-2000 (minimum weighing value 0.01g). The results were as follows:

1st	2000.01g
2nd	2000.00g
3rd	1999.98g
4th	1999.99g
5th	2000.02g

Repeatability

	Measured Value	Minimum Weighing Value (digit)
1st	2000.01g	1 dig
2nd	2000.00g	0 dig
3rd	1999.98g	-2 dig
4th	1999.99g	-1 dig
5th	2000.02g	2 dig
Mean	**2000.00g**	**0 dig**

As the minimum weighing value of GX-2000 is 0.01g,

1 digit = 0.01g

$$\sigma_{n-1} = \sqrt{\frac{\sum(\text{Measured Value} - \text{Mean})^2}{\text{Number of Measurements} - 1}}$$

Measured Value - Mean	(Measured Value -Mean)^2
0.01g	0.0001
0.00g	0.0000
-0.02g	0.0004
-0.01g	0.0001
0.02g	0.0004

$$\sum(\text{Measured Value} - \text{Mean})^2 = 0.0010$$

$$\frac{\sum(\text{Measured Value} - \text{Mean})^2}{\text{Number of Measurements} - 1} = 0.00025$$

$$\sigma_{n-1} = \sqrt{\frac{\sum(\text{Measured Value} - \text{Mean})^2}{\text{Number of Measurements} - 1}} = 0.01588 = 0.016g$$

Repeatability

Alternatively,

Measured Value - Mean	(Measured Value -Mean)^2
1 dig	1
0 dig	0
-2 dig	4
-1 dig	1
2 dig	2

$$\sum(\text{Measured Value} - \text{Mean})^2 = 10$$

$$\frac{\sum(\text{Measured Value} - \text{Mean})^2}{\text{Number of Measurements} - 1} = 2.5$$

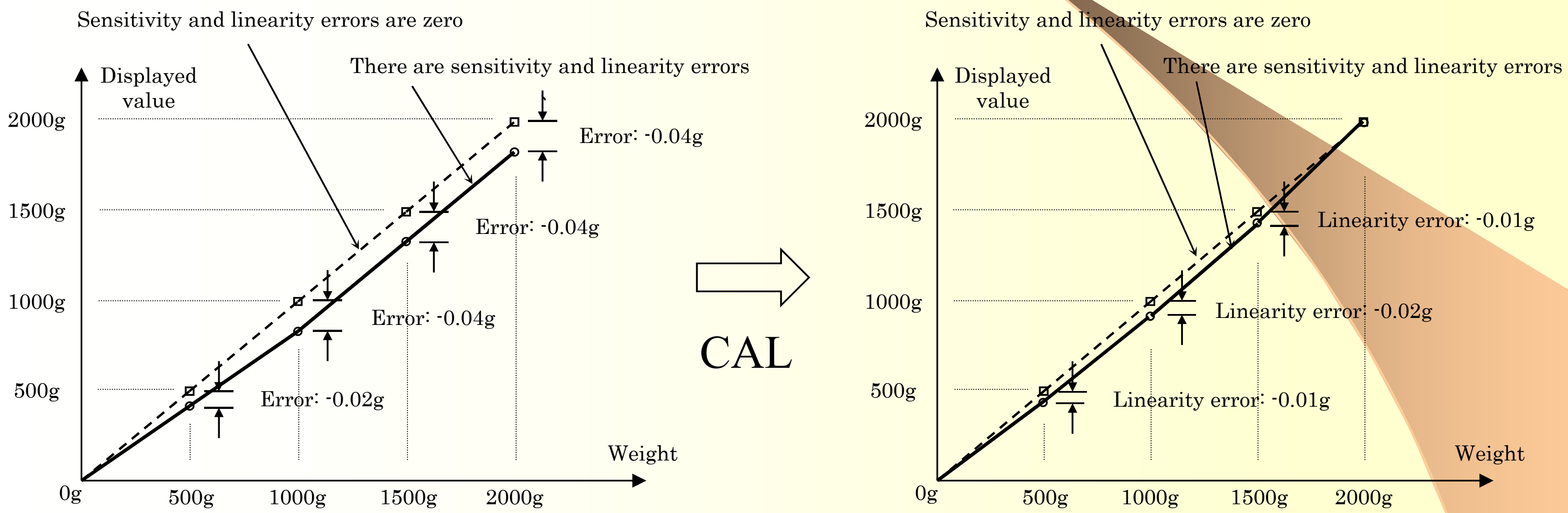
$$\sigma_{n-1} = \sqrt{\frac{\sum(\text{Measured Value} - \text{Mean})^2}{\text{Number of Measurements} - 1}} = 1.588 \fallingdotseq 1.6dig = 0.016g$$

Since the repeatability of GX-2000 is 0.01g, the performance of this balance would be **unsatisfactory**.

Linearity

- Linearity refers to the deviation in the measured value from the ideal (linear) curve between 0g and the maximum capacity.
- It is necessary to correct the slope of the measurement curve by conducting a calibration first, because the weighing results will vary depending on the acceleration of gravity at the point of use or can be influenced by temperature changes.

Linearity



- When calibrated with a 2000g weight, the measurement curve will be corrected by –0.04g at the point of 2000g.
- Likewise, it will be corrected by –0.01g, -0.02g, and –0.03g at 500g, 1000g, and 1500g respectively. The maximum linearity error after the calibration will thus be –0.02g (at 1000g).
- Since the specification of linearity for GX-2000 is ±0.02g, the performance is judged to be satisfactory in this case.

Sensitivity Drift

- Sensitivity drift is the increase or decrease in the measured value that occurs when the same mass is weighed at different ambient temperatures.
- The rate of variation is expressed in “ppm (Parts Per Million).”
- For example, the sensitivity drift of GX-2000 is ±2ppm/ºC. This means that the measured value can vary by up to “±2/1,000,000” with a 1ºC change in temperature.

Sensitivity Drift

Ex 1) We calibrated GX-2000 so that it would display 2000.00g at 20ºC. If the ambient temperature later became 30ºC, what would be the maximum error of measurement caused by sensitivity drift?

$$\text{Measured Value (Referential Value)} \times (\text{ppm}/^{\circ}\text{C}) \times \text{Variation in Temperature}$$

$$\begin{aligned}
&2000.00(g) \times \{\pm 2(\text{ppm}/^{\circ}\text{C})\} \times \{30(^{\circ}\text{C}) - 20(^{\circ}\text{C})\} \\
&= 2000.00 \times \frac{\pm 2}{1000000} \times 10 \\
&= \pm 0.04(g)
\end{aligned}$$

This indicates that there can be a variation of up to ±0.04g (1999.96 ~ 2000.04g) from the initial value of 2000.00g when the temperature changes by 10ºC.

Sensitivity Drift

Ex 2) The following results were obtained using GX-2000. What is the sensitivity drift (ppm/ºC)?

Weight	Ambient Temperature	Measured Value
2000.00g	20ºC	2000.00g
	30ºC	1999.97g

$$\left(\frac{\text{Result B (after change)} - \text{Result A (before change)}}{\text{Result A (before change)}}\right) \times 1000000 = (\text{ppm})$$

$$\frac{\text{ppm}}{\text{Variation in Temperature}} = (\text{ppm}/^{\circ}\text{C})$$

$$\frac{1999.97 - 2000.00}{2000.00} \times 1000000 = -15(\text{ppm})$$

$$\frac{-15}{30 - 20} = -1.5(\text{ppm}/^{\circ}\text{C})$$

This satisfies the specification of sensitivity drift for GX-2000, which is ±2ppm/ºC.